Newtonian Theory for the Compression Surface of Airfoils at Moderate or Large Incidence

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This paper considers the problem of steady Newtonian flow over the compression surface of sharp-edged, two-dimensional thin airfoils at moderate or large angles of attack. By combining the Newtonian limits $\gamma \to 1$ and $M_\infty \to \infty$ (where γ is the ratio of the specific heats of the gas and M_∞ the freestream Mach number) with a geometric limiting process in which the wing thickness approaches zero, a system of approximate equations giving a first-order correction to the Newtonian flow is found. The equations are solved in closed form and a simple formula for the coefficient of surface pressure is obtained. The formula shows the effects of nonzero $(\gamma - 1)$ and finite values of M_∞ as well as the surface curvature and angle of attack. Results are presented for several airfoils at various flow parameters.

Introduction

THE problem of steady two-dimensional flow at high Mach numbers has received great attention over the past decade. In particular, the case of thin pointed-nose airfoils at small angles of attack and attached shock waves has been investigated extensively. The shock wave formed below the lower surface of the wing is not weak because of its high Mach number and, therefore, the linearized supersonic flow theory and its modification (Busemann's theory) are not applicable. Thus, more accurate methods such as the tangent wedge or shock expansion[1] method must be tried. Although these methods can be useful at moderate and large angles of attack, they are lengthy and time consuming and do not have sufficient theoretical background.

One of the most rigorous high Mach number theories for two-dimensional flow past thin airfoils at small angles of attack is the hypersonic small-disturbance theory.[2] Although the equations have been greatly simplified in this theory, they are still nonlinear and far from being solved with any kind of generality. A possible exception to this is the Newtonian theory of Cole[3] derived from hypersonic small-disturbance equations. However, Cole's theory is applicable only to thin airfoils at small angles of attack. It is a zero-order theory showing the effect of wing surface curvature and does not show the effects of nonzero $(\gamma - 1)$ or finite values of M_∞. Although Cole has derived first-order approximation equations that can show these effects, he has not solved them.

In contrast to the case of small angles of attack, problems of moderate or large angles of attack have received very little attention. Practically speaking, however, the case of large angles of attack is the one about which more information is needed. This is most evident from Orlik-Ruckemann's[4] survey of aircraft needs and capabilities. This survey emphasizes the need for information about both the in- and out-of-phase components of the damping derivatives in pitching oscillations at high Mach numbers. It is quite obvious that such information cannot be obtained by, for example, the perturbation method without first considering the steady flow problem carefully.

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Because of the slow progress in developing suitable analytical methods, numerical methods have been used extensively in current research to solve flow problems. The method of characteristics and finite-difference approximations are used for supersonic and hypersonic flow problems. Those methods are exact and powerful, but they require massive amounts of computer storage and time. Also, they do not give enough insight into the physics of the flow and are not suitable for extensive parametric studies.

In this paper, we propose a first-order Newtonian theory for the flow past the compression surface of thin airfoils at moderate or large angles of attack. By combining the Newtonian limits $\gamma \to 1$ and $M_\infty \to \infty$ with a geometric limiting process in which the thickness of the airfoil approaches zero while the angle of attack remains fixed, we derive a system of equations and boundary conditions that can be solved easily in a closed form. Thus, a simple formula for the coefficient of surface pressure showing the effect of all of the parameters appearing in the full problem is found. The formula is then used to present results for several airfoils at various flight conditions.

The Perturbation Theory

Consider a two-dimensional airfoil of length $\bar{\ell}$ and small thickness (the order of magnitude of the thickness will be given later) and sharp leading edge to be placed in a supersonic stream of a gas at an angle of attack α that could be large. Take the $\bar{x}$ and $\bar{y}$ axes as shown in Fig. 1 (physical quantities will be denoted by a bar). Assume that α, γ, and M_∞ are such that the shock wave formed below the lower surface is attached to the apex of the wing. Consider the flow over the lower surface only (the compression surface). Denote by $\bar{u}$ and $\bar{v}$ the velocity components in the $\bar{x}$ and $\bar{y}$ directions, respectively, and by $\bar{p}$ and $\bar{\rho}$ the pressure and density of the gas in the flowfield of interest bounded by the lower surface of the airfoil and the shock wave. The steady, nonviscous, nonheat-conducting, perfect-gas equations of continuity, momentum, and energy are given by

$$(\bar{\rho}\bar{u})_{\bar{x}} + (\bar{\rho}\bar{v})_{\bar{y}} = 0 \tag{1a}$$

$$\bar{u}\bar{u}_{\bar{x}} + \bar{v}\bar{u}_{\bar{y}} + \frac{1}{\bar{\rho}}\bar{p}_{\bar{x}} = 0 \tag{1b}$$

$$\bar{u}\bar{v}_{\bar{x}} + \bar{v}\bar{v}_{\bar{y}} + \frac{1}{\bar{\rho}}\bar{p}_{\bar{y}} = 0 \tag{1c}$$

$$\bar{u}\left[\bar{p}_{\bar{x}}-\left(\frac{\gamma\bar{p}}{\bar{\rho}}\right)\bar{\rho}_{\bar{x}}\right]+\bar{v}\left[\bar{p}_{\bar{y}}-\left(\frac{\gamma\bar{p}}{\bar{\rho}}\right)\bar{\rho}_{\bar{y}}\right]=0 \tag{1d}$$

Subscripts here and in what follows denote partial derivatives. Let the equation of the lower surface be

$$B(\bar{x},\bar{y})=\bar{y}-\bar{F}(\bar{x})=0 \tag{2}$$

Assume that the function $\bar{F}(\bar{x})$ is given. The boundary condition to be satisfied at the wing surface is that the gas velocity be tangential to the surface, that is,

$$\bar{q}\cdot\nabla B=0, \quad \nabla=\frac{\partial}{\partial\bar{x}}\bar{i}+\frac{\partial}{\partial\bar{y}}\bar{j} \tag{3}$$

where $\bar{q}$ is the velocity vector at the wing surface and $\bar{i}$ and $\bar{j}$ the unit vectors in the $\bar{x}$ and $\bar{y}$ directions, respectively. Let the shock wave be given by

$$S(\bar{x},\bar{y})=\bar{y}-\bar{S}(\bar{x})=0 \tag{4}$$

where $\bar{S}(\bar{x})$ is an unknown function to be determined as part of the solution. The conditions to be satisfied at the shock are the Rankine-Hugoniot jump conditions given by

$$[\bar{\rho}(\bar{q}\cdot\nabla S)]=0 \tag{5a}$$

$$[\bar{\rho}(\bar{q}\cdot\nabla S)^2+(\nabla S)^2\bar{p}]=0 \tag{5b}$$

$$\left[\frac{1}{2}(\bar{q}\cdot\nabla S)+\frac{\gamma}{(\gamma-1)}\frac{\bar{p}}{\bar{\rho}}(\nabla S)^2\right]=0 \tag{5c}$$

$$[\bar{q}\times\nabla S]=0 \tag{5d}$$

The square brackets denote the change in the enclosed quantity across the shock. Equations (5a-5c) are the usual conservation equations of mass, momentum, and specific enthalpy across a shock wave, while Eq. (5d) is a vector equation equivalent to a scalar equation expressing the conservation of tangential velocity.

It is well known that the limits of $\gamma\to1$ and $M_\infty\to\infty$, applied to Eqs. (1), (3), and (5), with the body $\bar{F}(\bar{x})$ and the angle of attack α fixed, lead to the Newtonian limiting values. To derive a first-order correction to the Newtonian flow, the above limits will be associated with a geometric limiting process in which the wing thickness approaches zero and the angle of attack remains fixed. Thus, a small parameter ϵ is defined by

$$\epsilon=\frac{\gamma-1}{\gamma+1}+\frac{2}{(\gamma+1)M_\infty^2} \tag{6}$$

Physically, ϵ is the density ratio across a normal shock wave. It is very interesting to observe that regardless of the value of γ, $\epsilon<1$ for all the range of supersonic flow, 1 for sonic flow, and >1 for the entire range of subsonic flow. Thus, ϵ can be used as another alternative to M_∞ in classifying flowfields as being subsonic, sonic, or supersonic. Therefore, seeking a series solution in powers of ϵ for supersonic flow (not necessarily Newtonian flow) should, essentially, be the equivalent to seeking a series solution in powers of $1/M_\infty^2$. Notice that ϵ reduces to $1/M_\infty^2$ in the special case of $\gamma=1$. It is obvious that ϵ gets very close to zero for Newtonian flow. Further, assume that the $\bar{y}$ coordinate in the flowfield of interest is of order $0(\epsilon)$ and define new independent variables as

$$x=\bar{x}/\bar{\ell}, \quad y=\bar{y}/\bar{\ell}\epsilon\tan\alpha \tag{7}$$

where y is assumed to remain of order 0(1) in the limit $\epsilon\to0$. The appearance of $\tan\alpha$ in the definition of y is not essential, but is a convenience by which we obtain a particularly simple form of the approximate equations to follow. We now propose the following asymptotic series expansions for Newtonian flow, as $\epsilon\to0$:

$$\frac{\bar{u}(\bar{x},\bar{y})}{U_\infty}=\cos\alpha+\epsilon\cos\alpha\,u(x,y)+0(\epsilon^2) \tag{8a}$$

$$\frac{\bar{v}(\bar{x},\bar{y})}{U_\infty}=\epsilon\sin\alpha\,v(x,y)+0(\epsilon^2) \tag{8b}$$

$$\frac{\bar{p}(\bar{x},\bar{y})-P_\infty}{\rho_\infty U_\infty^2}=\sin^2\alpha+\epsilon\sin^2\alpha\,p(x,y)+0(\epsilon^2) \tag{8c}$$

$$\frac{\rho_\infty}{\bar{\rho}(\bar{x},\bar{y})}=a\epsilon-a\epsilon^2\rho(x,y)-0(\epsilon^3) \tag{8d}$$

$$\bar{F}(\bar{x})=\bar{\ell}\epsilon\tan\alpha\,F(x) \tag{8e}$$

$$\bar{S}(\bar{x})=\bar{\ell}\epsilon\tan\alpha[F(x)+S(x)]+0(\epsilon^2) \tag{8f}$$

where a is a constant to be fixed later and the functions in the right-hand sides of Eqs. (8) are assumed to remain of order 0(1) as $\epsilon\to0$. Equations (8) are expected to give reasonable approximation of the flow for small values of ϵ. This is the situation with the Newtonian flow considered here. Nonuniformity might be expected as $\epsilon\to1$. This is the case of low supersonic flow. For such values of ϵ, further terms of the series might be needed. It is to be noted, however, that ϵ is small for a wide range of M_∞ and γ. For example, when M_∞ is as low as 2.0 and γ is as high as 1.2, $\epsilon=0.32$; when $M_\infty=2.5$ and $\gamma=1.4$, $\epsilon=0.3$. It is known in nonlinear perturbation theories[5] that the leading term in the series is the most important one and that it shows all of the essential features of the flow. Other terms in the series add only to the accuracy. Equations (8e) and (8f) show that both the body thickness and the shock wave thickness are of order $0(\epsilon)$. It follows that the shock layer thickness is also of order $0(\epsilon)$. Therefore, the present approximation may be considered as a thin shock layer theory.

Substituting Eqs. (8) into Eqs. (1), (3), and (5) and retaining the lowest order terms in ϵ results in

$$a=\frac{1+\sin^2\alpha N}{\sin^2\alpha(1+N)} \tag{9}$$

Then, we obtain the following system I of equations and boundary conditions:

$$v_y=0 \tag{10a}$$

$$u_x+vu_y=0 \tag{10b}$$

$$v_x+ap_y=0 \tag{10c}$$

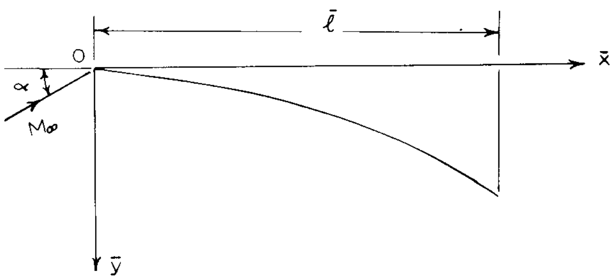

Fig. 1 Wing lower surface geometry and coordinate system.

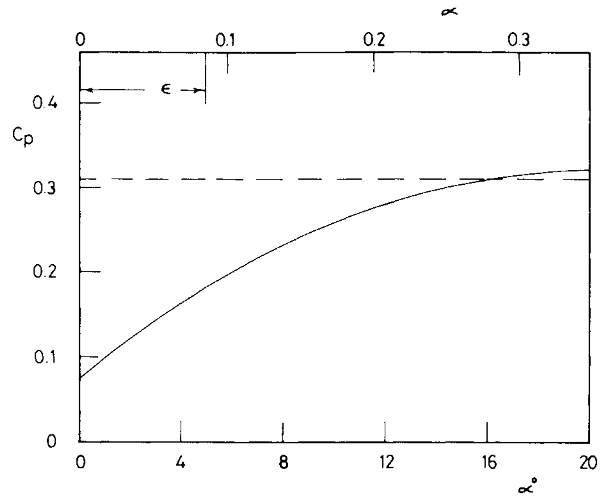

Fig. 2 Comparison for C_p for a class of wedges with a 20 deg deflection angle, $\gamma = 1.1$, $M_\infty = 5.0$ [—— Eq. (18), ----- exact].

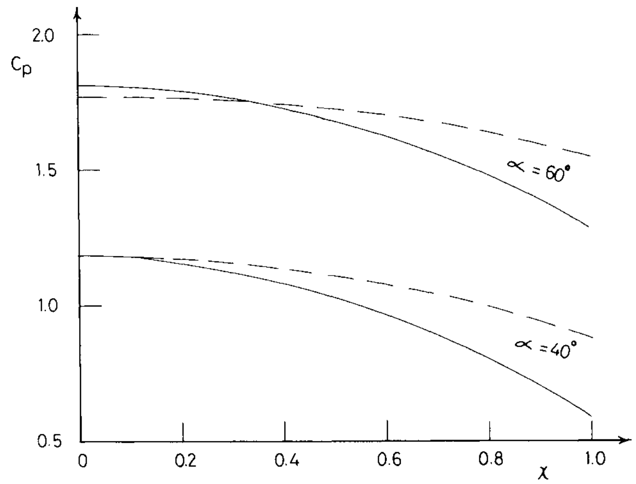

Fig. 3 Comparison for C_p for the concave airfoil, $\bar{F}(\bar{x}) = \bar{\ell}[0.129x - 0.05(x^3 - x)]$, $\gamma = 1.0$, $M_\infty = \infty$ [—— Eq. (18), ----- Newton-Busemann].

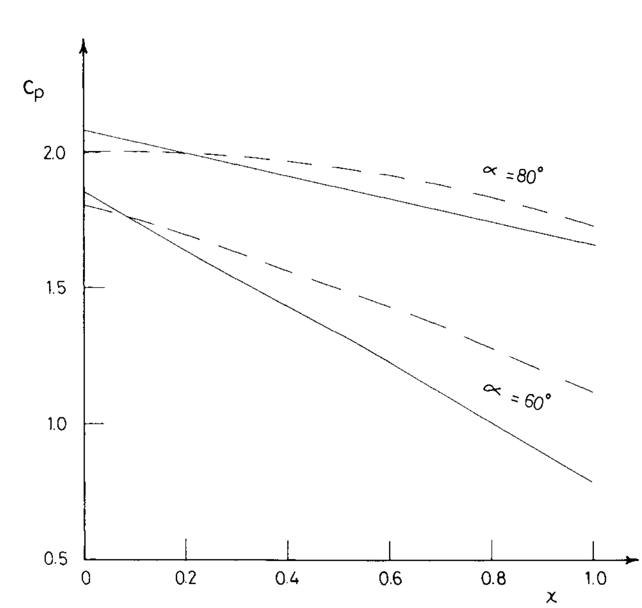

Fig. 4 Comparison for C_p for the lower surface of a biconvex symmetric circular airfoil with thickness ratio 10%, $\gamma = 1.0$, $M_\infty = \infty$ [—— Eq. (18), ----- Newton-Busemann].

$$\left(\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)(p - \rho) = 0 \tag{10d}$$

at

$$y = F(x), \quad v = F'(x) \tag{11}$$

at

$$y = F(x) + S(x)$$

$$p = 2[F'(x) + S'(x)] - \frac{N}{(1+N)} - \frac{1}{\sin^2\alpha(1+N)} \tag{12a}$$

$$\rho = \frac{1}{(1+\sin^2\alpha N)}\left\{2[F'(x) + S'(x)] + \frac{N}{1+N}\right\} \tag{12b}$$

$$v = F'(x) + S'(x) - \frac{N}{(1+N)} - \frac{1}{\sin^2\alpha(1+N)} \tag{12c}$$

$$u = -\tan^2\alpha[F'(x) + S'(x)] \tag{12d}$$

where

$$N = \left(\frac{\gamma - 1}{2}\right)M_\infty^2$$

It is seen that system I gives a first-order correction to the Newtonian flow past the compression surface of airfoils, provided the shock wave is attached to the apex of the wing and ϵ is small. It is interesting to observe that all the parameters appearing in the full problem are retained in system I. It is also of interest that, although Eqs. (10) are nonlinear, they are quite simple. Note that the unknown

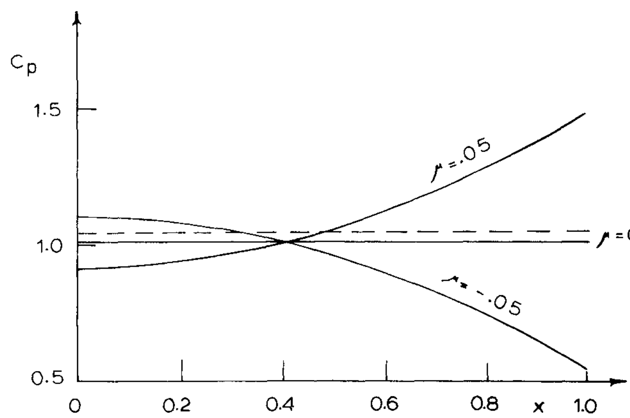

Fig. 5 Variation of C_p with x for a class of airfoils $\bar{F}(\bar{x}) = \bar{\ell}[0.129x + \mu(x^3 - x)]$, $\alpha = 35$ deg, $\gamma = 1.1$, $M_\infty = 5.0$ [—— Eq. (18), ----- exact].

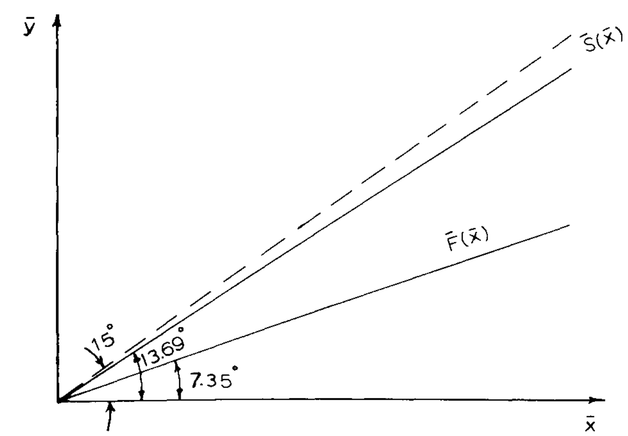

Fig. 6 Wedge airfoil and shock wave at $\alpha = 35$ deg, $\gamma = 1.1$, $M_\infty = 5.0$ [—— Eq. (18), ----- exact].

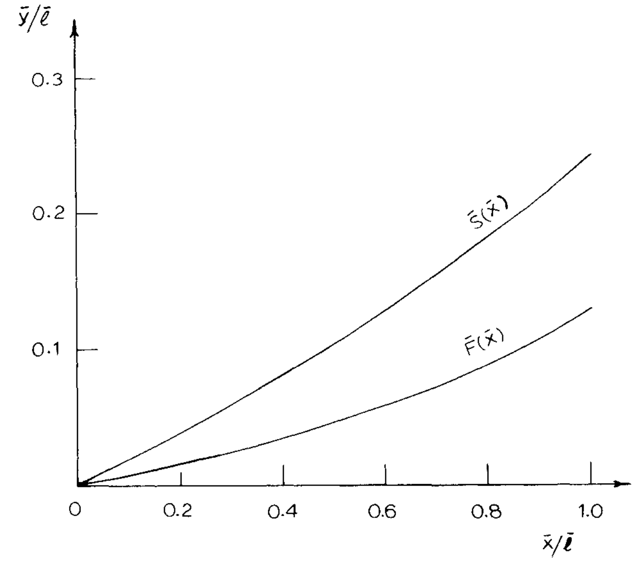

Fig. 7 Concave airfoil $\bar{F}(\bar{x})=\bar{\ell}[0.129x+0.05(x^3-x)]$ and shock wave, $\gamma=1.1$, $\alpha=35$ deg, $M_\infty=5.0$.

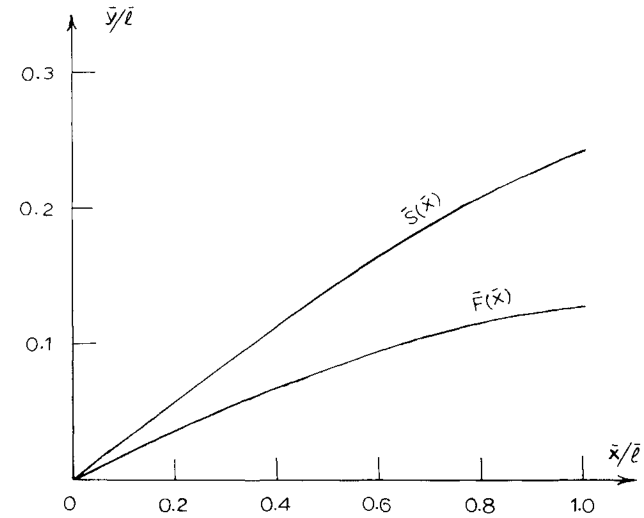

Fig. 8 Convex airfoil $\bar{F}(\bar{x})=\bar{\ell}[0.129x-0.05(x^3-x)]$ and shock wave, $\gamma=1.1$, $\alpha=35$ deg, $M_\infty=5.0$.

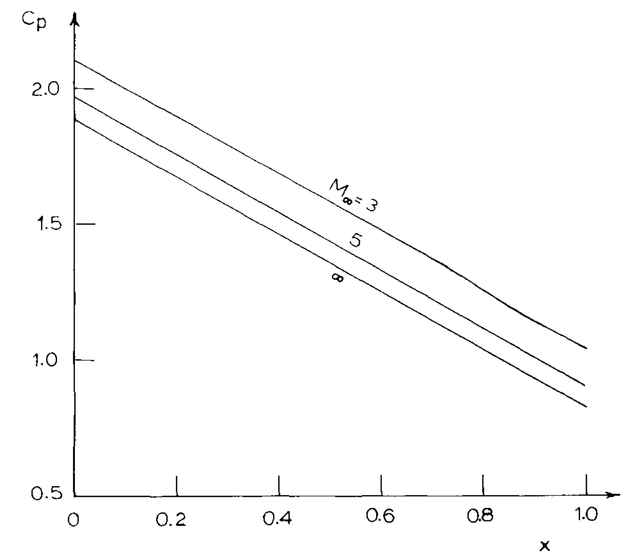

Fig. 9 Variation of C_p with x for the lower surface of a symmetric biconvex circular airfoil with thickness ratio 10%, $\gamma=1.05$, and $\alpha=60$ deg.

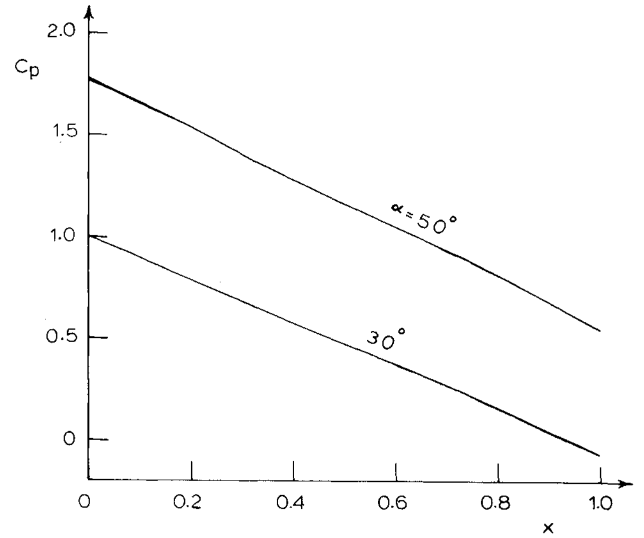

Fig. 10 Variation of C_p with x for the lower surface of a symmetric biconvex circular airfoil with thickness ratio 10%, $\gamma=1.15$, and $M_\infty=4.0$.

functions v, u, and p, are decoupled and can be found successively rather than simultaneously. Finally, it should be noted that a basic assumption in deriving system I is that the angle of attack α remains fixed as $\epsilon\to0$. Therefore, system I can be expected to give good results for only moderate or large angles of attack. A nonuniformity should be expected if α and ϵ have the same orders of magnitude. This latter case should be treated in a different way. (Cole's Newtonian theory already gives a zero-order approximation in this case.)

It should also be noted that Messiter[6] has used the limits $\gamma\to1$ and $M_\infty\to\infty$ with α fixed to derive a first-order correction to the Newtonian flow past delta wings. He used as a perturbation parameter the quantity

$$\frac{\gamma-1}{\gamma+1}+\frac{2}{(\gamma+1)M_\infty^2\sin^2\alpha}$$

The dependence of the small parameter above on α is inconvenient. The parameter ϵ used here is more suitable because it is independent of α and is less than one for the entire range of supersonic flow.

Solution of System I

Because system I is quite simple, its solution presents no difficulty and can be found as follows. Equation (10a) shows that, to the present degree of approximation, the function v is independent of y and that v and the shock wave function S can be found by equating values of v at the wing surface and shock wave. Doing so, we get

$$v(x,y)=F'(x) \tag{13a}$$

and

$$S'(x)=a \tag{13b}$$

Equation (13b) again shows that within the present approximation the function S is linear in x. Therefore, the shock wave slope differs from the body slope by just a constant and does not have the same curvature at the same x. Using Eq. (13a) in Eqs. (10b) and (10c) they become

$$u_x+F'(x)u_y=0 \tag{14a}$$

$$p_y=-(1/a)F''(x) \tag{14b}$$

The general solution of Eqs. (14) is given by

$$u(x,y)=G[y-F(x)] \tag{15a}$$

$$p(x,y)=-(1/a)F''(x)y+h(x) \tag{15b}$$

The functions G and h are arbitrary, to be fixed by the values of u and p at the shock wave. Thus, substituting Eqs. (12a) and (12d) into Eqs. (15), the latter become

$$u(x,y)=-\tan^2\alpha\left\{a+F'\left[\frac{y-F(x)}{a}\right]\right\} \tag{16a}$$

$$p(x,y)=-\frac{1}{a}F''(x)[y-F(x)-ax]+2[F'(x)+a] -\frac{(1+N\sin^2\alpha)}{\sin^2\alpha(1+N)} \tag{16b}$$

Using a similar procedure, the function ρ can be found from Eqs. (10d) and (12b), but it is of no interest in evaluating the pressure forces unless one wants to go to a higher approximation. The function ρ will be ignored in what follows.

Having found the function p, the coefficient of surface pressure C_p can be found. First, the surface pressure is given by

$$p[x,F(x)]=xF''(x)+2F'(x)+\frac{1+\sin^2\alpha N}{\sin^2\alpha(1+N)} \tag{17}$$

and C_p defined as usual by

$$C_p=\frac{p[x,F(x)]-P_\infty}{\frac{1}{2}\rho_\infty U_\infty^2}$$

will be

$$C_p=2\sin^2\alpha\left\{1+\epsilon\left[xF''(x)+2F'(x)+\frac{1+\sin^2\alpha N}{\sin^2\alpha(1+N)}\right]\right\} \tag{18}$$

It is seen that Eq. (18) for C_p is quite simple and includes all the parameters appearing in the full problem. It can be used to find C_p on the lower surface of an airfoil of small thickness, provided that the bow shock is attached to the apex of the wing, ϵ is small, and $\alpha \gg \epsilon$.

Results

In Fig. 2, Eq. (18) for C_p is compared with the exact value for a class of wedges having a deflection angle equal to 20 deg. α varies from small values of order $0(\epsilon)$ (ϵ is shown on the figure) to values $\gg\epsilon$. It is seen that, when $\alpha\gg\epsilon$ (the correct range of use of the theory), the agreement is very good. Figure 3 compares the present method with the Newton-Busemann pressure law for the convex airfoil, $\bar{F}(\bar{x})=\ell[0.129x-0.05(x^3-x)]$ at $\gamma=1.0$ and $M_\infty=\infty$ and with $\alpha=40$ and 60 deg. The agreement is shown to be good and increases as α increases. In Fig. 4, the comparison is made with Newton-Busemann pressure law for the lower surface of a symmetric biconvex circular airfoil with thickness ratio equal to 10% and with $\gamma=1.0$ and $M_\infty=\infty$, at $\alpha=60$ and 80 deg. Again, the agreement is shown to be good and improves as α increases.

Figure 5 shows the variation of C_p with x for a class of airfoils having the same length-to-thickness ratio given by $\bar{F}(\bar{x})=\ell[0.129x+\mu(x^3-x)]$, where μ is a parameter. $\mu=0$ results in a wedge, $\mu>0$ gives concave airfoils, and $\mu<0$ gives convex airfoils. The exact value for the wedge flow is also shown in Fig. 5. The parameters α, γ, and M_∞ are respectively 35 deg, 1.1, and 5.0. The comparison for the wedge shows that the approximate value is very good, with an error less than 3%. Unfortunately, no results are available for comparison of the curved surface airfoils when $\gamma>1$ and $M_\infty<\infty$. Figure 6 shows that, for a wedge with semivertex angle of 7.35 deg at $\alpha=35$ deg, $\gamma=1.1$, and $M_\infty=5.0$, the shock wave angle is 13.69 deg and the exact value given by the shock wave relations is 15 deg. Again, the error in shock slope is about 9%. In Figs. 7 and 8, concave and convex airfoils are shown together with the shock waves. In Fig. 9, the variation of C_p with x for the lower surface of a biconvex symmetric airfoil with a thickness ratio of 10% is shown for various values of M_∞ at $\alpha=60$ deg and $\gamma=1.05$. In Fig. 10, the variation in C_p for the same airfoil at $\gamma=1.15$ and $M_\infty=4.0$ is shown for $\alpha=30$ and 50 deg. In Figs. 9 and 10, C_p varies almost linearly with x due to the small slope and curvature of the circular arc airfoil considered.

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